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Simulatietool 1.0

Het effect van internationale schokken doorheen het globale productienetwerk op de lokale economie

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Nederlandstalige samenvatting

Dit beleidsrapport situeert zich binnen de recurrente taken van de economische analyse unit van ECOOM zijnde ECOOM-STORE. Meer specifiek maakt dit rapport deel uit van pijler D "Internationale waardeketens". De focus van deze pijler ligt op het bestuderen van de mate waarin de Vlaamse economie geïntegreerd is in de globale waardeketens, oftewel de 'global value chains (GVC)', en welke mogelijke voor- en nadelen dit met zich mee kan brengen.

Een belangrijk aspect hierbinnen is het uitbouwen van een simulatietool (1.0)[1] waarbij de effecten van recente maar ook toekomstige economische schokken op de internationale handelsstromen in kaart gebracht worden. Enkele voorbeelden zoals de COVID-19 pandemie, de energiecrisis en recent de stijgende spanningen in het conflict tussen Israël en Hamas toonden aan dat er niet enkel directe effecten maar ook indirecte effecten zijn doorheen het globale productienetwerk op de Vlaamse economie. Om dit totaal effect te bestuderen, maakt dit beleidsrapport gebruik van de wereld input-outputtabellen.

De **belangrijkste resultaten** van dit rapport zijn:

1. Het **uitwerken van de methodologie** van de simulatietool. Concreet zijn er **vier verschillende benaderingen** die gebruikt kunnen worden om economische schokken te simuleren, zijnde (1) de Leontief vraag gedreven benadering, (2) de Ghosh aanbod gedreven benadering, (3) de hypothetische extractie - Leontief en (4) de hypothetische extractie – Ghosh.

2. Afhankelijk van het **soort economische schok**, zal een specifieke benadering meer of minder geschikt zijn om de economische impact in te schatten. Iedere benadering heeft specifieke voor- en nadelen. Bij de implementatie van de simulatietool is het daarom van belang dat de beleidsmakers en de onderzoekers **in onderling overleg** toekomstige beleidsvragen **vertalen** naar de bijhorende codering in de simulatietool.

3. Een **toepassing** waarbij de simulatietool geïllustreerd wordt aan de hand van een fictieve energieschok. De resultaten worden visueel gepresenteerd in Europese (of wereld) kaarten en/of staafdiagrammen waarbij de heterogene impact op de verschillende landen en sectoren zichtbaar wordt.

[1] De benaming simulatietool 1.0 verwijst naar een simulatietool op basis van wereld input-outputtabellen die gebruik maakt van data op land-sectorniveau. Conditioneel op het verkrijgen van toegang tot de B2B dataset zou ECOOM-STORE in de toekomst ook een simulatietool 2.0 willen uitwerken met de B2B data om zo schokken doorheen het Vlaamse productienetwerk te kunnen simuleren met Belgische bedrijfsdata. Verder is het de bedoeling om ook de huidige simulatietool verder te ontwikkelen na deze versie 1.0.

Simulation Tool: An Input-Output Approach

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Abstract:
This manuscript investigates the utilization of Input-Output Analysis to model economic shocks within production networks. Section 1 and 2 serve as an introduction, providing an initial exploration of the foundational elements of Input-Output Analysis. The subsequent section conducts a thorough review of three primary methodologies: (1) Leontief, (2) Ghosh, and (3) Hypothetical Extraction. Section 4 employs a stylized Input-Output Table to demonstrate these methodologies. Using the most recent OECD Inter-Country Input-Output (ICIO) Tables, Section 5 exemplifies the modeling of an energy shock from the Russian Federation to the European Union. Section 6 concludes.

Keywords: Input-Output Analysis, Production Networks, Global Value Chains

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1 Introduction

The analytical framework known as Input-Output Analysis, developed by Prof. Wassily Leontief in the late 1930s and distinguished with the Nobel Prize in Economic Science in 1973, has undergone a transformative evolution. Originally confined to scrutinizing domestic economies, contemporary iterations of Input-Output Analysis have expanded to encompass a broader dataset, offering a more nuanced examination of the interdependencies characterizing the global economy. Globalization has been shaped significantly by (i) technological advancements in information and communication, (ii) the relaxation of cross-border trade restrictions facilitated by international organizations (WTO, EU, NAFTA), and (iii) geopolitical shifts such as the dissolution of the USSR and the ascent of Asia.

A consequential outcome of these developments is the progressive disintegration of production processes across international borders. Corporations increasingly adopt a global perspective, outsourcing elements of production to specialized entities in foreign and often geographically distant nations. This paradigm shift has engendered increasingly intricate supply chains, resulting in a departure from the conventional notion of goods originating from a singular country, now replaced by the more accurate characterization of products as ”Made in the World.”

However, the COVID-19 pandemic has introduced a nuanced perspective on globalization. The heightened specialization of supply chains has rendered the global production network susceptible to localized disruptions, with reverberating consequences across the world. Consequently, policymakers are reevaluating their stance on globalization, with a discernible trend toward calls for increased autonomy and state intervention. Geopolitical events, such as Russia’s invasion of Ukraine and the European Union’s strategic decision to cease importing Russian oil and gas, underscore the critical need for understanding industrial and country interdependencies.

Against this backdrop the ’Simulation Tool’ was developed. This tool, conceived as a sophisticated yet accessible instrument, facilitates the expeditious assessment of shocks to specific industries and countries. Its relevance is particularly pronounced in the context of the European Union, given its intricate cross-country production network.

2 Model

In the standard open Input-Output Model Leontief (1944) interprets production as a function of final demand, given the technology used by each sector. The model yields equilibrium results, which satisfies optimality conditions. Each industry produces one homogeneous commodity and uses one homogeneous technology which determines the proportions of inputs used. The production technology is crucial for the interrelation of the sectors, it governs the exchange of goods used as inputs between each sector. Exogenous variables, namely final demand and value added, determine the level of activity in the economy given the economic structure. Each industry has a distinctive production technology, obeys strict constant returns to scale and allows for no substitution between inputs.

Ghosh (1958) builds upon the work by Leontief, but solves production as a function of value added instead. All other assumptions and functional forms remain the same as in the standard open IO model. The crucial difference lies in the interpretation. Instead of final demand driving production in the economy, value added (i.e. labour supply) generates production.

Equation (1) and (2) represent the two sides of production in the IO model[1]:

$$\boldsymbol{Zi} + \boldsymbol{f} = \boldsymbol{x} \tag{1}$$

$$\boldsymbol{i'Z} + \boldsymbol{v'} = \boldsymbol{x'} \tag{2}$$

where:

n ... number of countries
s ... number of industries/sectors
i, j ... index of country-industry pair, of which there are n*s
$\boldsymbol{Z}$ ... intermediate demand, $ns \times ns$ square matrix
$\boldsymbol{f}$ ... final demand, $ns \times 1$ (column) vector
$\boldsymbol{x}$ ... output as sectoral revenue, $ns \times 1$ (column) vector
$\boldsymbol{v'}$ ... value added, $1 \times ns$ (row) vector
$\boldsymbol{x'}$ ... output as sectoral expenditure, $1 \times ns$ (row) vector
$\boldsymbol{i}$ ($\boldsymbol{i'}$) ... sum matrix which sums rows (columns) of $\mathbf{Z}$
$\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{v}}$ ... diagonal matrix with output/value added along the main diagonal

On the one hand, the IO model is demand driven; a change in final demand is immediately followed by an increase/decrease of output. Put differently, output is infinitely elastic with respect to final demand; neither scarce factors of production nor sticky prices impede any adjustments needed to reach an equilibrium and satisfy final demand (Leontief, equation 1).
On the other hand, the IO model is supply driven as total revenues are determined by generating value added. Output is infinitely elastic with respect to factor revenues, as such, intermediate and final goods consumers absorb as much output as producers offer in order to reach an equilibrium (Ghosh, equation 2).

Notice that both equations are independent from each other, but can be linked when output becomes factor incomes and, conversely, when value added is transformed into final demand. Both final demand and value added are exogenous in the IO model, therefore, transforming one into another is also an exogenous process. In contrast to general equilibrium models, equations (1) and (2) cannot be solved simultaneously. Any modern version of the general equilibrium model means determining two vectors at the same time, one for prices and one for quantities, which are dual to one another. This is not the case in IO models and Leontief solves the open demand model through independent processes. Ghosh (1958) shows that applying a similar procedure yields a solution for a supply driven model.

2.1 Solving the IO Model

In a first step both equations above are rewritten in proportions (coefficients):

$$\boldsymbol{Ax} + \boldsymbol{f} = \boldsymbol{x} \tag{3}$$

$$\boldsymbol{x'B} + \boldsymbol{v'} = \boldsymbol{x'} \tag{4}$$

In the Leontief model, $\boldsymbol{A} = \boldsymbol{Z}\ \hat{\boldsymbol{x}}^{-1} = [\,a_{ij}\,] = [\,z_{ij}/x_j\,]$ is commonly referred as the *technical* coefficient matrix. Its elements show the proportion of each good i that each industry j uses as input to produce its homogeneous good.[2] Conversely, $\boldsymbol{B} = \hat{\boldsymbol{x}}^{-1}\ \boldsymbol{Z} = [\,b_{ij}\,] = [\,z_{ij}/x_i\,]$, shows

[1]Letters in bold denote matrices.
[2]In the case of a Multi-regional IO table i, j are country-industry pairs.

which proportion each industry i sells to every other industry j out of its total output.[3] The coefficients a_{ij} and b_{ij} are proportions of the sectoral expenditure (x_j) and sectoral revenue (x_i) respectively. Notice that $\boldsymbol{A}$ is technically determined, whereas $\boldsymbol{B}$ is not. From the viewpoint of the producer it is reasonable to assume that technology determines the selection and proportions of inputs, while there is no clear theoretical explanation for the amount or proportions of output producers sell to consumers. Furthermore, it is irrelevant to the producer whether her good is used as intermediate input or final demand good.

In fact both matrices, $\boldsymbol{A}$ and $\boldsymbol{B}$, are very similar under close inspection.[4] This is because both originate from two similar production models. Both matrices transform the space of produced goods into a space of produced goods, however, by different means: intermediate consumption (demand) or the distribution of inputs (supply). Furthermore, the sum of each column of $\boldsymbol{A}$ and each row of $\boldsymbol{B}$ are less than unity. This is because each industry uses goods as inputs in a smaller value than they produce and at the same time, the value of total supply of each produced good is larger than the value of goods absorbed by other producers as inputs. As a result, the economic system produces a surplus, either in the form of final demand goods or as value added.

Solving equations (3) and (4) determines: (i) the level of output necessary to satisfy exogenous final demand and (ii) the level of output necessary to generate the exogenous level of value added.

$$(\boldsymbol{I} - \boldsymbol{A})^{-1}\, \boldsymbol{f} = \boldsymbol{L}\, \boldsymbol{f} = \boldsymbol{x} \tag{5}$$

$$\boldsymbol{v}'\, (\boldsymbol{I} - \boldsymbol{B})^{-1} = \boldsymbol{v}'\, \boldsymbol{G} = \boldsymbol{x}' \tag{6}$$

$\boldsymbol{L} = [\, l_{ij}\,]$ is called the Leontief multiplier; its entries show the total requirements (direct + indirect) of inputs produced by industry i per unit of output produced by industry j to attain final demand $\boldsymbol{f}$. Conversely, $\boldsymbol{G} = [\, g_{ij}\,]$ show the total sales (direct + indirect) that industry j needs to encourage to every other industry i such that value added $\boldsymbol{v}'$ is attainable.

Finally, both the demand-driven and the supply-driven model determine output. However, we have not explicitly solved for prices as prices and quantities are not dual. The price model which has been accepted for long (Miller and Blair, 2009) assumes that the price level in each industry depends on the direct and indirect costs of primary inputs ($\boldsymbol{v}$), subject to the technology used in the system as a whole ($\boldsymbol{A}$).

$$(\boldsymbol{I} - \boldsymbol{A}')^{-1} = \boldsymbol{p} \tag{7}$$

Recall, that this equation can be solved independently from equation (5) because prices and quantities are not dual in the IO model.

2.2 Shocking the IO Model

What happens when final demand changes in one sector (f_k) in Leontief's demand-driven model? As immediate response, output changes in that sector proportionally and in turn, induces changes in the demand for inputs of the sector in question. In another round, production of the industries which supply inputs to the industries supplying to the affected industry adjust production as well. Continuing in this fashion, it is expected that output changes in every sector. According to the Leontief multiplier ($\boldsymbol{L}$), resources are available at every moment to support any

[3]Extracting the coefficients from the example economy below (Table 1) yields $a_{RU-GAS,EU-GAS} = 0.2$ and $b_{RU-GAS,EU-GAS} = 0.2$. The interpretation in the Leontief model would be that the European gas industry uses 20% of its inputs from the Russian gas industry. The interpretation in the Ghosh model would be that the Russian gas industry sells 20% of its intermediate output to the European gas industry.

[4]Matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are square, positive semi-definite, non-singular and share the associated eigenvalues.

level of production induced by final demand. Implicitly technical coefficients, $\boldsymbol{A}$, are assumed to stay the same, independent of the change in final demand. Therefore, the assumption of constant returns to scale and no substitution between inputs is essential, otherwise multipliers would be impossible to compute.

In Ghosh's supply-driven model a positive shock to factor incomes of one sector (v_k) would ripple through the system in a similar fashion, however, by inducing changes downstream of the affected sector instead of upstream as in the demand-driven model. As immediate response, the affected sector would proportionally increase its output, therefore, expanding producers' revenues in order to afford the extra amounts of inputs required to increase their own production. In the next round, the extended output of the originally affected industry induces other sectors to expand their own output. Continuing in this fashion, no industry faces difficulties to hire the extra factors of production or find consumers willing to pay for the additional production.[5]

The demand-driven and supply-driven model are jointly stable in the case of a uniform shock across all industries. This means that a $\Delta\%$ shock to $\boldsymbol{f}$ and $\boldsymbol{v}'$ leads to the same new output vector ($\boldsymbol{x}_1 = \boldsymbol{x}'_1$) and exchange matrix ($\boldsymbol{Z}_1$).

3 Methodology

There are a wide variety of measures to capture forward and backward linkages of a sector to the rest of the economy. Three approaches are commonly found in the literature to estimate the effects of output changes in a single or a subset of sectors: (i) final demand-driven, (ii) output-based and (iii) hypothetical extraction. There is no *best* method available for all policy evaluations, such that each method can be appropriate for specific settings.

The building block, the final demand (supply) driven approach has already been discussed in the previous section. In general, these methods are recommended for industries with a high ratio of final demand (value added) to output.[6] Otherwise, altering the final demand (value added) of a single industry will not sufficiently lower total industry output, as inter-industry exchanges are comparatively high.[7] Therefore, in sectors with a low final demand (value added) to output ratio, solely adjusting final demand (value added) would significantly underestimate the effects of an output shock in the industry.

The output-based approach generally starts by defining the outputs of the specific sector in question and outputs from related sectors. From there, adjustments are made to avoid double counting of sectoral output. Original multipliers are applied to the adjusted outputs to arrive at the economy-wide impact. The double counting problem is inherent in all methods using output as a starting point, and simply adjusting the outputs in the sectors does not solve the problem entirely.

Hypothetical extraction of a sector is the third major approach in the literature. Some variations of the method exist, see Groenewold and Madden (1987), however, all approaches still suffer from double-counting. In a related paper, Leung and Pooley (2002) extend the hypothetical extraction method for the supply driven Ghosh model. They argue that this allows them

[5]Prices are simply a residual of the model and do not feature as a constraint.

[6]These are usually industries producing final consumption goods and services, for example, passenger car manufacturers.

[7]Inter-industry exchange can only be altered by adjusting final demand of other related industries.

to better capture the effects of a change in the supply of output of an industry. The alternative formulation has the advantage to capture forward linkages instead of backward linkages, which might be a more appropriate measure for some industries and shocks. For example, a reduction in output due to a strike which affects primarily the customers of the affected industry.

In the following sections we will discuss the different methods to shock the IO model in much more detail. Notice that the IO model is a linear transformation, therefore, we can simply use a 1% shock and multiply by the desired magnitude of the shock afterwards. We use a *fictional* scenario to model a supply shock of Russian energy exports to the European Union (EU).

3.1 Leontief Demand Driven

In the standard demand-driven IO model (Leontief, 1944), the only option is to shock the exogenous final demand for Russian energy in the EU. However, given that energy is a crucial input in many industries, it seems very unlikely that we can induce a full supply stop of energy from Russia by solely adjusting the final demand in the energy sector itself. In order to alleviate this problem to some extent we could adjust the final demand in other industries which are highly dependable on Russian energy. However, this approach requires exceptional knowledge of inter-industry linkages and assumes that one can gauge the induced final demand changes relatively well.

Solving for the new exchange matrix by reducing final demand of Russian energy in the EU by 1% is straightforward. Let the Russia energy exports to the EU be given by sector k then the new final demand column vector $\boldsymbol{f}_1$ writes:

$$\boldsymbol{f}_1 = \begin{bmatrix} 1 & \dots & 1 & 0.99 & 1 & \dots & 1 \end{bmatrix} * \boldsymbol{f} = \begin{bmatrix} f_1 \\ \vdots \\ 0.99\, f_k \\ \vdots \\ f_n \end{bmatrix}$$

which gives rise to the new output (revenue) vector $\boldsymbol{x}_1$ by solving:

$$\boldsymbol{x}_1 = (\boldsymbol{I} - \boldsymbol{A})^{-1}\, \boldsymbol{f}_1 = \boldsymbol{L}\, \boldsymbol{f}_1$$

yielding the new exchange matrix $\boldsymbol{Z}_1$ via:

$$\boldsymbol{Z}_1 = \boldsymbol{A}\, \hat{\boldsymbol{x}}_1$$

The total (direct + indirect) output change induced by a reduction in Russian energy exports to the EU can be directly computed using the sectoral Leontief multipliers ($l_{i,j}$) and the change in final demand:

$$losses_{EU} = \Delta x_{EU} = \sum_k l_{EU,k}\; 0.01\; f_k$$

Using Leontief coefficients includes spillover effects via inter-industry linkages, hence, the figure represents sectoral losses after a new equilibrium is reached. However, by expanding the infinite geometric series of the Leontief inverse matrix we can can also compute the sectoral direct and indirect effects for each round of adjustment:

$$\boldsymbol{L} = (\boldsymbol{I} - \boldsymbol{A})^{-1} = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \boldsymbol{A}^3 + \dots$$

$$\boldsymbol{x}_1 = (\boldsymbol{I} - \boldsymbol{A})^{-1}\, \boldsymbol{f}_1 = \boldsymbol{I}\, \boldsymbol{f}_1 + \boldsymbol{A}\, \boldsymbol{f}_1 + \boldsymbol{A}^2\, \boldsymbol{f}_1 + \boldsymbol{A}^3\, \boldsymbol{f}_1 + \dots$$

Let $\Delta \boldsymbol{f}_1 = \boldsymbol{f}_1 - \boldsymbol{f}$, then the first round effect is given by $\Delta \boldsymbol{f}_1$, the second round effect by $\boldsymbol{A}\ \Delta \boldsymbol{f}_1$, the third round effect by $\boldsymbol{A}^2\ \Delta \boldsymbol{f}_1$ and so forth. We can further disseminate the effects across sectors, such that the output loss per industry i at round $r \in \{1, 2, 3, \ldots\}$ is given by:

$$loss_{EU,i,r} = \Delta x_{EU,i,r} = \boldsymbol{A}^{r-1}_{EU,i}\ \Delta \boldsymbol{f}$$

Where $\boldsymbol{A}^{r-1}_{EU,i}$ denotes the row of $\boldsymbol{A}^{r-1}$ corresponding to industry i of the EU.

3.2 Ghosh Supply Driven

The analysis in the Ghosh model follows the same procedure as in the Leontief model, however, using the appropriate matrices for the supply-driven IO model instead. Notice we can formulate the model in changes right away. Consequently, a negative shock of 1% to value added can be written as:

$$\Delta \boldsymbol{v}' = \begin{bmatrix} 0 \\ \vdots \\ -0.01 \\ \vdots \\ 0 \end{bmatrix} * \boldsymbol{v}' = \begin{bmatrix} 0 & \ldots & -0.01\ v_k & \ldots & 0 \end{bmatrix} = \begin{bmatrix} 0 & \ldots & \Delta v_k & \ldots & 0 \end{bmatrix}$$

the changes to the corresponding new output (expenditure) vector $\Delta \boldsymbol{x}'_1$ are:

$$\Delta \boldsymbol{x}' = \Delta \boldsymbol{v}'\ (\boldsymbol{I} - \boldsymbol{B})^{-1} = \Delta \boldsymbol{v}'\ \boldsymbol{G}$$

which yields the changes to the exchange matrix $\Delta \boldsymbol{Z}$ via:

$$\Delta \boldsymbol{Z} = \Delta \hat{\boldsymbol{x}}'\ \boldsymbol{B}$$

Consequently, the total (direct + indirect) change in output induced by a reduction in Russian energy exports to the EU is simply given by the sectoral Ghosh multiplier ($g_{i,j}$) and the change in value added:

$$losses_{EU} = \Delta x_{EU} = \sum_k g_{EU,\,k}\ \Delta v_k$$

Using the coefficients of the Ghosh multiplier ($\boldsymbol{G}$) includes all inter-industry spillover effects, hence, the losses represent sectoral losses after a new equilibrium is reached. By expanding the infinite geometric series we can measure direct and indirect effects for each round of adjustment:

$$\boldsymbol{G} = (\boldsymbol{I} - \boldsymbol{B})^{-1} = \boldsymbol{I} + \boldsymbol{B} + \boldsymbol{B}^2 + \boldsymbol{B}^3 + \ldots$$

$$\Delta \boldsymbol{x}' = \Delta \boldsymbol{v}'\ (\boldsymbol{I} - \boldsymbol{B})^{-1} = \Delta \boldsymbol{v}'\ \boldsymbol{I} + \Delta \boldsymbol{v}'\ \boldsymbol{B} + \Delta \boldsymbol{v}'\ \boldsymbol{B}^2 + \Delta \boldsymbol{v}'\ \boldsymbol{B}^3 + \ldots$$

Hence, the first round effect is given by $\Delta \boldsymbol{v}'_1$, the second round effect by $\boldsymbol{B}\ \Delta \boldsymbol{v}'_1$, the third round effect by $\boldsymbol{B}^2\ \Delta \boldsymbol{v}'_1$ and so forth. We can further disseminate the effects across sectors, such that the output loss per industry i at round $r \in \{1, 2, 3, \ldots\}$ is given by:

$$loss_{EU,i,r} = \Delta x_{EU,i,r} = \Delta \boldsymbol{v}'\ \boldsymbol{B}^{r-1}_{EU,i}$$

Where $\boldsymbol{B}^{r-1}_{EU,i}$ denotes the column of $\boldsymbol{B}^{r-1}$ corresponding to industry i of the EU.

3.3 Hypothetical extraction: Leontief Demand Driven

Hypothetical extraction methods exploit the linearity of the IO model to exogenise the output of a subset of sectors. As above, we suppose an exogenous shock to affect gross output of the Russian energy exports to the EU (sector k). By partitioning the IO model into affected and unaffected sectors, we assume that some of the sectoral outputs are fixed, i.e. all other sectors $i \neq k$. Thereby, we can asses the impact of changes in outputs of the exogenised sector (k) on outputs of the endogenous sectors in the economy ($i \neq k$), as well as on the final demand of the exogenized sector (k). The partitioned demand driven model can be written as:

$$\begin{bmatrix} \boldsymbol{x}_k \\ \boldsymbol{x}_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{kk} & \boldsymbol{A}_{ki} \\ \boldsymbol{A}_{ik} & \boldsymbol{A}_{ii} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_k \\ \boldsymbol{x}_i \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}_k \\ \boldsymbol{f}_i \end{bmatrix}$$

By partitioning the matrix we effectively split the economy into (i) the exogenous variables: Russian energy output to the EU ($\boldsymbol{x}_k$) and final demand in other industries ($\boldsymbol{f}_i$); and (ii) the endogenous variables: output in all other sectors ($\boldsymbol{x}_i$) and final demand of Russian energy in the EU ($\boldsymbol{f}_k$). Writing out the respective lines of the partitioned matrix yields the following:

$$\boldsymbol{x}_k = \boldsymbol{A}_{kk}\, \boldsymbol{x}'_k + \boldsymbol{A}_{ki}\, \boldsymbol{x}'_i + \boldsymbol{f}_k \tag{8}$$

$$\boldsymbol{x}_i = \boldsymbol{A}_{ik}\, \boldsymbol{x}'_k + \boldsymbol{A}_{ii}\, \boldsymbol{x}'_i + \boldsymbol{f}_i \tag{9}$$

The shock affects output of sector k itself, hence, its gross output is no longer determined by its final demand. Consequently, $\boldsymbol{x}_k$ is no longer determined endogenously and equation (8) ceases to hold. To solve for an exogenous change in output, $\Delta\boldsymbol{x}_k$, we only consider the second line of the matrix. Equation (9) represents the market clearing condition for the unaffected sectors ($i \neq k$); their output ($\boldsymbol{x}_i$) depends on the exogenous variables $\boldsymbol{x}_k$ and $\boldsymbol{f}_i$. We can rewrite equation (9) in terms of changes:

$$\Delta\boldsymbol{x}_i = (\boldsymbol{I} - \boldsymbol{A}_{ii})^{-1}\, (\boldsymbol{A}_{ik}\, \Delta\boldsymbol{x}'_k + \Delta\boldsymbol{f}_i)$$

Notice that this measure is incomplete when dealing with a less than total reduction of sector k. A less-than-total reduction in industry k will have an impact on industry k itself via intra-industry linkages and the associated inter-industry links to other sectors of the economy. Consequently, the multiplier would underestimate the potential impact of a less-than-total reduction in output of sector k on the economy.

By assuming no additional exogenous change in final demand in the unaffected sectors ($\Delta\boldsymbol{f}_i = 0$) simplifies the expression to:

$$\Delta\boldsymbol{x}_i = (\boldsymbol{I} - \boldsymbol{A}_{ii})^{-1}\, \boldsymbol{A}_{ik}\, \Delta\boldsymbol{x}'_k \tag{10}$$

The matrix $(\boldsymbol{I} - \boldsymbol{A}_{ii})^{-1}\, \boldsymbol{A}_{ik}$ measures the backward linkage effects of a change in output of a sector ($\Delta\boldsymbol{x}_k$) on all other sectors in the economy ($\Delta\boldsymbol{x}_i$). Reformulating equation (10) and (8), allows us to recover the implied changes to final demand of the affected sector ($\boldsymbol{f}_k$):

$$\Delta\boldsymbol{f}_k = (\boldsymbol{I} - \boldsymbol{A}_{kk})\, \Delta\boldsymbol{x}'_k - \boldsymbol{A}_{ki}\, (\boldsymbol{I} - \boldsymbol{A}_{ii})^{-1}\, \boldsymbol{A}_{ik}\, \Delta\boldsymbol{x}'_k$$

As in the original demand driven approach, it is assumed that the exogenous change in output ($\Delta\boldsymbol{x}_k$) does not affect the direct requirement matrix ($\boldsymbol{A}$) of the economy. In other words, production technologies of every sector in the economy remain unchanged. In addition, by extending the infinite geometric series in equation (10) we can obtain the effects for each round of adjustment:

$$\Delta\boldsymbol{x}_i = (\boldsymbol{I} - \boldsymbol{A}_{ii})^{-1}\, \boldsymbol{A}_{ik}\, \Delta\boldsymbol{x}'_k = (\boldsymbol{I} + \boldsymbol{A}_{ii} + \boldsymbol{A}_{ii}^2 + \boldsymbol{A}_{ii}^3 + \ldots)\, \boldsymbol{A}_{ik}\, \Delta\boldsymbol{x}'_k$$

We can further disseminate the effects across sectors, such that the output loss per industry i at round $r \in \{1, 2, 3, \ldots\}$ is given by:

$$loss_{EU,i,r} = \Delta x_{EU,i,r} = \boldsymbol{A}_{ik}{}^{r-1}_{EU,i}\, \boldsymbol{A}_{ik}\, \Delta \boldsymbol{x}_k$$

Where $\boldsymbol{A}_{ik}{}^{r-1}_{EU,i}$ denotes the row of $\boldsymbol{A}_{ik}^{r-1}$ corresponding to industry i of the EU.

Papadas and Dahl (1999) argue that if a single sector is exogenised, a supply driven multiplier can be obtained via the sum of column vector $(\boldsymbol{I} - \boldsymbol{A}_{ii})^{-1}\ \boldsymbol{A}_{ik}$. By exogenising every sector one at a time, supply driven multipliers can be obtained for all industries in the economy. Furthermore, if output of sector k is not allowed to vary after an initial exogenous change (e.g. through a change due to final demand adjustments), the measure $(\Delta \boldsymbol{x}_i)$ is in fact appropriate, even in the case of a less-then-total reduction in sector k's output.

3.4 Hypothetical extraction: Ghosh Supply Driven

The above analysis provides the potential impact only from the backward linkage point of view. A similar framework can be extended to the Ghosh model to capture forward linkages effects, see Leung and Pooley (2002). The setup follows the same procedure as the demand driven model, instead using the output coefficient matrix ($\boldsymbol{B}$) and value added ($\boldsymbol{v'}$). By exogenising sector k, the Ghosh model can be partitioned as:

$$\begin{bmatrix}\boldsymbol{x}'_k & \boldsymbol{x}'_i\end{bmatrix} = \begin{bmatrix}\boldsymbol{x}'_k & \boldsymbol{x}'_i\end{bmatrix} \begin{bmatrix}\boldsymbol{B}_{kk} & \boldsymbol{B}_{ki} \\ \boldsymbol{B}_{ik} & \boldsymbol{B}_{ii}\end{bmatrix} + \begin{bmatrix}\boldsymbol{v}'_k & \boldsymbol{v}'_i\end{bmatrix}$$

and writing out the respective lines in changes yields:

$$\Delta \boldsymbol{x}'_k = \Delta \boldsymbol{x}'_k\, \boldsymbol{B}_{kk} + \Delta \boldsymbol{x}'_i\, \boldsymbol{B}_{ik} + \Delta \boldsymbol{v}'_k \tag{11}$$

$$\Delta \boldsymbol{x}'_i = \Delta \boldsymbol{x}'_k\, \boldsymbol{B}_{ki} + \Delta \boldsymbol{x}'_i\, \boldsymbol{B}_{ii} + \Delta \boldsymbol{v}'_i \tag{12}$$

To solve for an exogenous change in output, $\Delta \boldsymbol{x}'_k$, we only consider the second line of the matrix as sector k's gross output is no longer determined by its value added. Equation (12) represents the market clearing condition for the unaffected sectors ($i \neq k$); their output ($\boldsymbol{x}'_i$) depends on the exogenous variables $\boldsymbol{x}'_k$ and $\boldsymbol{v}'_i$. Assuming value added does not change in the endogenised sectors ($\Delta \boldsymbol{v}'_i = 0$), yields the change in output in sectors $i \neq k$ after the new equilibrium has been reached:

$$\Delta \boldsymbol{x}'_i = \Delta \boldsymbol{x}'_k\, \boldsymbol{B}_{ki}\, (\boldsymbol{I} - \boldsymbol{B}_{ii})^{-1} \tag{13}$$

The matrix $\boldsymbol{B}_{ik}\ (\boldsymbol{I} - \boldsymbol{B}_{ii})^{-1}$ measures the forward linkage effects of a change in output of a sector ($\Delta \boldsymbol{x}'_k$) on all other sectors in the economy ($\Delta \boldsymbol{x}'_i$). Reformulating equation (13) and (11), allows us to recover the implied changes to value added of the affected sector ($\boldsymbol{v}'_k$):

$$\Delta \boldsymbol{v}'_k = \Delta \boldsymbol{x}'_k\, (\boldsymbol{I} - \boldsymbol{B}_{kk}) - \Delta \boldsymbol{x}'_k\, \boldsymbol{B}_{ki}\, (\boldsymbol{I} - \boldsymbol{B}_{ii})^{-1}\, \boldsymbol{B}_{ik}$$

Per round adjustment effects are recovered as in the demand driven model. Hence, the output loss per industry i at round $r \in \{1, 2, 3, \ldots\}$ is given by:

$$loss_{EU,i,r} = \Delta x_{EU,i,r} = \Delta \boldsymbol{x}_k\, \boldsymbol{B}_{ki}\, \boldsymbol{B}_{ii}{}^{r-1}_{EU,i}$$

Where $\boldsymbol{B}_{ii}{}^{r-1}_{EU,i}$ denotes the column of $\boldsymbol{B}_{ii}^{r-1}$ corresponding to industry i of the EU.

4 Illustration

We illustrate the four methods in a numerical example via the economy given in Table 1.

Table 1: Numerical Example

		Intermediate Demand (z_{ij})						Final Demand (F_{ij})			
		EU		RU		US					
Country	Industry	Gas	Other	Gas	Other	Gas	Other	EU	RU	US	Tot. Rev. (x_i)
EU	Gas	100	500	50	150	100	400	300	50	200	1850
	Other	200	900	150	400	200	950	750	250	500	4300
RU	Gas	350	150	400	300	100	50	100	200	50	1700
	Other	550	400	800	900	50	150	200	500	100	3650
US	Gas	50	400	50	100	500	750	300	50	500	2700
	Other	150	750	150	300	900	1500	700	150	900	5500
	Value Added (v_i)	450	1200	100	1500	850	1700				
	Tot. expend. (x_i)	1850	4300	1700	3650	2700	5500				

Notes: Values are expressed in million euros.

The building blocks of the IO model are:

$$\boldsymbol{Z} = \begin{bmatrix} 100 & 500 & 50 & 150 & 100 & 400 \\ 200 & 900 & 150 & 400 & 200 & 950 \\ 350 & 150 & 400 & 300 & 100 & 50 \\ 550 & 400 & 800 & 900 & 50 & 150 \\ 50 & 400 & 50 & 100 & 500 & 750 \\ 150 & 750 & 150 & 300 & 900 & 1500 \end{bmatrix} \quad \boldsymbol{F} = \begin{bmatrix} 300 & 50 & 200 \\ 750 & 250 & 500 \\ 100 & 200 & 50 \\ 200 & 500 & 100 \\ 300 & 50 & 500 \\ 700 & 150 & 900 \end{bmatrix} \quad \boldsymbol{x} = \begin{bmatrix} 1850 \\ 4300 \\ 1700 \\ 3650 \\ 2700 \\ 5500 \end{bmatrix} \quad \boldsymbol{v} = \begin{bmatrix} 450 \\ 1200 \\ 100 \\ 1500 \\ 850 \\ 1700 \end{bmatrix}$$

The technical and output coefficient matrices, $\boldsymbol{A}$ and $\boldsymbol{B}$ are computed as:

$$\boldsymbol{A} = \boldsymbol{Z}\,\hat{\boldsymbol{x}}^{-1} = \begin{bmatrix} 0.1 & 0.1 & 0.0 & 0.0 & 0.0 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.1 & 0.2 \\ 0.2 & 0.0 & 0.2 & 0.1 & 0.0 & 0.0 \\ 0.3 & 0.1 & 0.5 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.0 & 0.0 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.3 & 0.3 \end{bmatrix}$$

$$\boldsymbol{B} = \hat{\boldsymbol{x}}^{-1}\,\boldsymbol{Z} = \begin{bmatrix} 0.1 & 0.3 & 0.0 & 0.1 & 0.1 & 0.2 \\ 0.0 & 0.2 & 0.0 & 0.1 & 0.0 & 0.2 \\ 0.2 & 0.1 & 0.2 & 0.2 & 0.1 & 0.0 \\ 0.2 & 0.1 & 0.2 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.0 & 0.0 & 0.2 & 0.3 \\ 0.0 & 0.1 & 0.0 & 0.1 & 0.2 & 0.3 \end{bmatrix}$$

We immediately observe that the diagonal elements remain the same for both the demand- and supply-driven model. As noted above the other coefficients look similar too. Finally, the Leontief

($\boldsymbol{L}$) and Ghosh ($\boldsymbol{G}$) inverse are obtained via:

$$\boldsymbol{L} = (\boldsymbol{I} - \boldsymbol{A})^{-1} = \begin{bmatrix} 1.2 & 0.3 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.5 & 1.6 & 0.5 & 0.4 & 0.4 & 0.5 \\ 0.4 & 0.2 & 1.5 & 0.2 & 0.2 & 0.2 \\ 0.8 & 0.5 & 1.1 & 1.6 & 0.3 & 0.3 \\ 0.2 & 0.3 & 0.3 & 0.2 & 1.4 & 0.4 \\ 0.5 & 0.6 & 0.6 & 0.4 & 0.8 & 1.8 \end{bmatrix}$$

$$\boldsymbol{G} = (\boldsymbol{I} - \boldsymbol{B})^{-1} = \begin{bmatrix} 1.2 & 0.6 & 0.2 & 0.3 & 0.3 & 0.7 \\ 0.2 & 1.6 & 0.2 & 0.3 & 0.3 & 0.7 \\ 0.5 & 0.5 & 1.5 & 0.5 & 0.3 & 0.5 \\ 0.4 & 0.5 & 0.5 & 1.6 & 0.2 & 0.5 \\ 0.1 & 0.5 & 0.2 & 0.2 & 1.4 & 0.8 \\ 0.2 & 0.5 & 0.2 & 0.3 & 0.4 & 1.8 \end{bmatrix}$$

4.1 Leontief Demand Driven

Following the discussion above we are limited to introduce a shock into the Leontief demand-driven economy via a shock to final demand. We assume a 10% adverse shock to final demand for Russian gas in the EU, everything else remains constant. Therefore, the change in the final demand matrix $\boldsymbol{F}$ writes:

$$\Delta\boldsymbol{F} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -10 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \implies \Delta\boldsymbol{f} = \begin{bmatrix} 0 \\ 0 \\ -10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Consequently, the change in the total output vector writes:

$$\Delta\boldsymbol{x} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\,\Delta\boldsymbol{f} = \boldsymbol{L}\,\Delta\boldsymbol{f} = \begin{bmatrix} -2.1 \\ -5.0 \\ -15.2 \\ -11.2 \\ -2.5 \\ -5.7 \end{bmatrix}$$

This yields the change in the exchange matrix via:

$$\Delta\boldsymbol{Z} = \boldsymbol{A}\,\Delta\hat{\boldsymbol{x}} = \begin{bmatrix} -0.1 & -0.6 & -0.4 & -0.5 & -0.1 & -0.4 \\ -0.2 & -1.1 & -1.3 & -1.2 & -0.2 & -1.0 \\ -0.4 & -0.2 & -3.6 & -0.9 & -0.1 & -0.1 \\ -0.6 & -0.5 & -7.2 & -2.8 & -0.0 & -0.2 \\ -0.1 & -0.5 & -0.4 & -0.3 & -0.5 & -0.8 \\ -0.2 & -0.9 & -1.3 & -0.9 & -0.8 & -1.6 \end{bmatrix}$$

The change in value added is given by the residual:

$$\Delta\boldsymbol{v}' = \Delta\boldsymbol{x}' - \boldsymbol{i}'\Delta\boldsymbol{Z} = \begin{bmatrix} -0.5 \\ -1.4 \\ -0.9 \\ -4.6 \\ -0.8 \\ -1.8 \end{bmatrix}$$

Notice, we can compute the total (direct + indirect) output reduction for the EU directly using the Leontief multipliers:

$$
\begin{aligned}
loss_{EU} &= \Delta x_{EU} \\
&= l_{EU-GAS,RU-GAS}\ \Delta f_{RU-GAS} + l_{EU-OTHER,RU-GAS}\ \Delta f_{RU-GAS} \\
&= 0.21 * (-10) + 0.50 * (-10) \\
&= -(2.1 + 5.0) = -7.1
\end{aligned}
\tag{14}
$$

Furthermore, we differentiate between the direct and indirect effects by expanding the infinite geometric series of the Leontief inverse matrix such that:

Table 2: Per Round Adjustment

Round		Computation	$\Delta \boldsymbol{x}_r$
1	direct	$\boldsymbol{I}\ \Delta \boldsymbol{f} = \begin{bmatrix} 0 \\ 0 \\ -10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$= \begin{bmatrix} 0 \\ 0 \\ -10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$
2	indirect	$\boldsymbol{A}\ \Delta \boldsymbol{f} = \begin{bmatrix} 0.1 & 0.1 & 0.0 & 0.0 & 0.0 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.1 & 0.2 \\ 0.2 & 0.0 & 0.2 & 0.1 & 0.0 & 0.0 \\ 0.3 & 0.1 & 0.5 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.0 & 0.0 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.3 & 0.3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$= \begin{bmatrix} -0.3 \\ -0.9 \\ -2.4 \\ -4.7 \\ -0.3 \\ -0.9 \end{bmatrix}$
3	indirect	$\boldsymbol{A}^2\ \Delta \boldsymbol{f} = \begin{bmatrix} 0.0 & 0.1 & 0.0 & 0.0 & 0.0 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.0 & 0.0 \\ 0.2 & 0.1 & 0.2 & 0.1 & 0.1 & 0.1 \\ 0.0 & 0.1 & 0.0 & 0.0 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.2 & 0.2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$= \begin{bmatrix} -0.5 \\ -1.1 \\ -1.0 \\ -2.5 \\ -0.5 \\ -1.1 \end{bmatrix}$
4	indirect	$\boldsymbol{A}^3\ \Delta \boldsymbol{f} = \quad \vdots$	$= \begin{bmatrix} -0.4 \\ -0.9 \\ -0.6 \\ -1.4 \\ -0.5 \\ -1.0 \end{bmatrix}$
		$\vdots$	

We clearly see that there is no direct effect of a reduction in European final demand from Russia on European producers itself, the entirety of the burden falls on Russian producers. However, Russian producers require fewer inputs from the EU (and the US), as they supply less final goods to the EU such that European production suffers in a second round of adjustments. This procedure continues until a new equilibrium is reached. Notice, how the shock travels upstream, it connects producers with their suppliers, hence, the name *backward linkages*.

Our aim is to get some estimate about potential consequences of a energy supply shock from Russia to the EU. We can only adjust final demand of energy from Russia in the EU in the demand-driven approach. A negative 10% shock leads to a reduction of output in the EU by only 7.1 million euros, a 0.10% reduction from initial total output in the EU. The reason for this is that final demand of gas from Russia is not sufficient to induce a major change in the exchange matrix ($\boldsymbol{Z}$), as Russian gas functions as an intermediate input instead of a final consumption good.

4.2 Ghosh Supply Driven

In the supply-driven IO model we are instead restricted to a shock on value added, i.e. the payment sector of the economy. We view this as a reduction in labour supply in Russia by 10%, for example, following a worker strike or a pandemic induced closure in some energy producing firms. Consequently, the change in the value added vector can be expressed by:

$$\Delta \boldsymbol{v} = \begin{bmatrix} 0 \\ 0 \\ -10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The change in total output is computes as:

$$\Delta \boldsymbol{x}' = \Delta \boldsymbol{v}' \, (\boldsymbol{I} - \boldsymbol{B})^{-1} = \Delta \boldsymbol{v}' \, \boldsymbol{G} = \begin{bmatrix} -4.6 \\ -5.4 \\ -15.2 \\ -5.2 \\ -2.8 \\ -5.0 \end{bmatrix}$$

This yields the change in the exchange matrix via:

$$\Delta \boldsymbol{Z} = \Delta \hat{\boldsymbol{x}} \, \boldsymbol{B} = \begin{bmatrix} -0.2 & -1.2 & -0.1 & -0.4 & -0.2 & -1.0 \\ -0.3 & -1.1 & -0.2 & -0.5 & -0.3 & -1.2 \\ -3.1 & -1.3 & -3.6 & -2.7 & -0.9 & -0.4 \\ -0.8 & -0.6 & -1.1 & -1.3 & -0.1 & -0.2 \\ -0.1 & -0.4 & -0.1 & -0.1 & -0.5 & -0.8 \\ -0.1 & -0.7 & -0.1 & -0.3 & -0.8 & -1.4 \end{bmatrix}$$

The change in final demand in the Ghosh model is the residual::

$$\Delta \boldsymbol{f} = \Delta \boldsymbol{x} - \Delta \boldsymbol{Z} \, \boldsymbol{i} = \begin{bmatrix} -1.4 \\ -1.9 \\ -3.1 \\ -1.1 \\ -0.9 \\ -1.6 \end{bmatrix}$$

Notice that the linearity of the IO model implies that the shares of final demand ($\boldsymbol{F}_s$) remains the same even after the shock to value added. Consequently, we can recover individual final demand changes via:

$$\boldsymbol{F}_s = \hat{\boldsymbol{f}}^{-1}\,\boldsymbol{F} \quad \Longrightarrow \quad \Delta\boldsymbol{F} = \Delta\hat{\boldsymbol{f}}\,\boldsymbol{F}_s = \begin{bmatrix} -0.7 & -0.1 & -0.5 \\ -0.9 & -0.3 & -0.6 \\ -0.9 & -1.8 & -0.4 \\ -0.3 & -0.7 & -0.1 \\ -0.3 & -0.1 & -0.5 \\ -0.6 & -0.1 & -0.8 \end{bmatrix}$$

As in the Leontief demand-driven model we can compute the adjustments per round by:

$$\Delta\boldsymbol{x}'_r = \Delta\boldsymbol{v}'\,\boldsymbol{B}^{r-1}$$

There is no direct effect of a reduction in Russian labour supply to European output in the first round. However, as Russian output declines, as less labour is available, Russia is unable to sell as much output (intermediate and final goods) to the EU. In turn, the EU, without the necessary inputs from Russia to sustain the initial output level also needs to reduce its output for both intermediate and final goods. Notice, how the shock travels downstream, it connects producers with buyers, hence the name *forward linkages*.

After a new equilibrium is reached European output falls by 10 million euros (0.15%) after a reduction of 10% in value added in Russia. In comparison, the shock to value added induces a much more severe adjustment in the EU compared to a shock of the same magnitude in the previous exercise. The reason for this, is that the value added shock severely hits the Russian economy with which the European economy is linked relatively tightly (compared to the US). As the goal is to obtain an estimate for a supply shock of Russian energy to the EU, this does slightly better, however, is still unsatisfactory. The main reason being that the sales from Russia to the EU remain relatively stable, and fall only because of the decline of Russian output itself but not via the direct channel of an export stop.

4.3 Hypothetical Extraction: Leontief Demand Driven

The hypothetical extraction method exploits the linearity of the IO model, partitioning the economy into an exogenous and endogenous part. This allows us to model a reduction of 10% in the total output of Russian gas sector directly. Without loss of generality we rearrange the matrices such that the exogenous sector occupies the first row and column. Therefore, the necessary building blocks are:

$$\Delta\boldsymbol{x}_k = \begin{bmatrix} -170 \end{bmatrix} \quad \boldsymbol{A}_{kk} = \begin{bmatrix} 0.2 \end{bmatrix} \quad \boldsymbol{A}_{ki} = \begin{bmatrix} 0.2 \\ 0.0 \\ 0.1 \\ 0.0 \\ 0.0 \end{bmatrix} \quad \boldsymbol{A}_{ik} = \begin{bmatrix} 0.0 \\ 0.1 \\ 0.5 \\ 0.0 \\ 0.1 \end{bmatrix} \quad \boldsymbol{A}_{ii} = \begin{bmatrix} 0.1 & 0.1 & 0.0 & 0.0 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.2 \\ 0.3 & 0.1 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.0 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.3 & 0.3 \end{bmatrix}$$

We assume that there is no additional shock to the exogenous final demand in the other countries such that $\Delta\boldsymbol{f}'_i = 0$ and which yields the endogenous change in output in the rest of the economy:

$$\Delta\boldsymbol{x}_i = (\boldsymbol{I} - \boldsymbol{A}_{ii})^{-1}\,\boldsymbol{A}_{ik}\,\Delta\boldsymbol{x}'_k = \begin{bmatrix} -23.2 \\ -30.4 \\ -50.8 \\ -34.4 \\ -68.5 \end{bmatrix}$$

We can also recover the endogenous change in final demand via:

$$\Delta \boldsymbol{f}_k = (\boldsymbol{I} - \boldsymbol{A}_{kk})\,\Delta \boldsymbol{x}'_k - \boldsymbol{A}_{ki}\,(\boldsymbol{I} - \boldsymbol{A}_{ii})^{-1}\,\boldsymbol{A}_{ik}\,\Delta \boldsymbol{x}'_k = \begin{bmatrix}-109.1\end{bmatrix}$$

With the endogenous changes in hand we can rebuild the economy:[8]

$$\Delta \boldsymbol{Z} = \begin{bmatrix} -5.5 & -5.8 & -5.9 & -4.2 & -1.3 & -0.6 \\ -0.7 & -1.6 & -19.8 & -2.1 & -1.3 & -5.0 \\ -2.0 & -3.3 & -35.6 & -5.6 & -2.6 & -11.8 \\ -10.9 & -9.1 & -15.8 & -12.5 & -0.6 & -1.9 \\ -0.7 & -0.8 & -15.8 & -1.4 & -6.4 & -9.3 \\ -2.0 & -2.5 & -29.7 & -4.2 & -11.5 & -18.7 \end{bmatrix} \qquad \Delta \boldsymbol{f} = \begin{bmatrix} 0.0 \\ 0.0 \\ -109.1 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

$$\Delta \boldsymbol{x} = \begin{bmatrix} -23.2 \\ -30.4 \\ -170.0 \\ -50.8 \\ -34.4 \\ -68.5 \end{bmatrix} \qquad \Delta \boldsymbol{v} = \begin{bmatrix} -1.4 \\ -7.4 \\ -47.4 \\ -20.9 \\ -10.8 \\ -21.2 \end{bmatrix}$$

Inspecting the changes in our economy we immediately spot that, in contrast to the previous two methods, there are substantial adjustments in the exchange matrix ($\Delta \boldsymbol{Z}$). The demand-driven model is unable to achieve such dramatic changes in intermediate goods flows with the same shock to final demand only. Hence, this is is major advantage of the hypothetical extraction method. A 10% output shock in the Russian energy sector leads to proportional reduction in exports of intermediate goods to the EU. Final goods exports of Russian gas to the EU decline by about 31% and overall output in the EU declined by 0.87%.

Nevertheless, the mechanism behind the hypothetical extraction method remains the same as in the demand-driven method. If we would use the output induced change in final demand ($\Delta f_k = -109.1$) in the demand-driven framework, we would obtain the same equilibrium as in the example above.

Finally, we can also follow how the change in output of the Russian economy travels through the production network until a new equilibrium is reached. The output adjustments of the EU and the US are given by:

$$\Delta \boldsymbol{x}_i = (\boldsymbol{I} - \boldsymbol{A}_{ii})^{-1}\,\boldsymbol{A}_{ik}\,\Delta \boldsymbol{x}'_k = (\boldsymbol{I} + \boldsymbol{A}_{ii} + \boldsymbol{A}_{ii}^2 + \boldsymbol{A}_{ii}^3 + \ldots)\,\boldsymbol{A}_{ik}\,\Delta \boldsymbol{x}'_k$$

which yields the following table of adjustments:

[8] All matrices are in the original order.

Table 3: Per Round Adjustment

Round		Computation	$\Delta \boldsymbol{x}_r$
1	direct	$\boldsymbol{I}\ \boldsymbol{A}_{ik}\ \Delta \boldsymbol{x}'_k = \boldsymbol{I} \begin{bmatrix} 0.0 \\ 0.1 \\ 0.5 \\ 0.0 \\ 0.1 \end{bmatrix} \Delta \boldsymbol{x}'_k =$	$\begin{bmatrix} -5.9 \\ -19.8 \\ -15.8 \\ -15.8 \\ -29.7 \end{bmatrix}$
2	indirect	$\boldsymbol{A}_{ii}\ \boldsymbol{A}_{ik}\ \Delta \boldsymbol{x}'_k = \begin{bmatrix} 0.1 & 0.1 & 0.0 & 0.0 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.2 \\ 0.3 & 0.1 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.0 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.3 & 0.3 \end{bmatrix} \boldsymbol{A}_{ik}\ \Delta \boldsymbol{x}'_k =$	$\begin{bmatrix} -7.3 \\ -4.6 \\ -13.7 \\ -8.1 \\ -16.8 \end{bmatrix}$
3	indirect	$\boldsymbol{A}_{ii}^2\ \boldsymbol{A}_{ik}\ \Delta \boldsymbol{x}'_k = \begin{bmatrix} 0.0 & 0.1 & 0.0 & 0.0 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.0 & 0.1 \\ 0.0 & 0.1 & 0.0 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.2 & 0.2 \end{bmatrix} \boldsymbol{A}_{ik}\ \Delta \boldsymbol{x}'_k =$	$\begin{bmatrix} -4.2 \\ -2.5 \\ -8.8 \\ -4.5 \\ -9.4 \end{bmatrix}$
4	indirect	$\boldsymbol{A}_{ii}^3\ \boldsymbol{A}_{ik}\ \Delta \boldsymbol{x}'_k =$	$\begin{bmatrix} -2.4 \\ -1.5 \\ -5.2 \\ -2.6 \\ -5.4 \end{bmatrix}$
		$\vdots$	

4.4 Hypothetical Extraction: Ghosh Demand Driven

The analysis in the supply-driven hypothetical extraction framework follows the same structure as above. Without loss of generality we rearrange the matrices such that the exogenous sector occupies the first row and column. The building blocks become:

$$\Delta \boldsymbol{x}_k = \begin{bmatrix} -170 \end{bmatrix} \quad \boldsymbol{B}_{kk} = \begin{bmatrix} 0.2 \end{bmatrix} \quad \boldsymbol{B}_{ki} = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.2 \\ 0.1 \\ 0.0 \end{bmatrix} \quad \boldsymbol{B}_{ik} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.2 \\ 0.0 \\ 0.0 \end{bmatrix} \quad \boldsymbol{B}_{ii} = \begin{bmatrix} 0.1 & 0.3 & 0.1 & 0.1 & 0.2 \\ 0.0 & 0.2 & 0.1 & 0.0 & 0.2 \\ 0.2 & 0.1 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.0 & 0.2 & 0.3 \\ 0.0 & 0.1 & 0.1 & 0.2 & 0.3 \end{bmatrix}$$

We assume that there is no additional shock to value added in the other countries such that $\Delta \boldsymbol{v}'_i = 0$. The endogenous change in output in the rest of the economy simplifies to:

$$\Delta \boldsymbol{x}'_i = \Delta \boldsymbol{x}'_k\ \boldsymbol{B}_{ki}\ (\boldsymbol{I} - \boldsymbol{B}_{ii})^{-1} = \begin{bmatrix} -21.7 \\ -21.2 \\ -34.9 \\ -27.6 \\ -71.3 \end{bmatrix}$$

We can also recover the endogenous change in value added of Russia via:

$$\Delta \boldsymbol{v}'_k = \Delta \boldsymbol{x}'_k\ (\boldsymbol{I} - \boldsymbol{B}_{kk}) - \Delta \boldsymbol{x}'_k\ \boldsymbol{B}_{ki}\ (\boldsymbol{I} - \boldsymbol{B}_{ii})^{-1}\ \boldsymbol{B}_{ik} = \begin{bmatrix} -109.1 \end{bmatrix}$$

With the endogenous changes in hand we can rebuild the economy:[9]

$$\Delta \boldsymbol{Z} = \begin{bmatrix} -5.1 & -4.5 & -1.9 & -3.8 & -1.3 & -0.6 \\ -0.6 & -1.1 & -5.7 & -1.7 & -1.1 & -4.6 \\ -5.9 & -7.9 & -35.6 & -15.8 & -7.9 & -37.6 \\ -7.6 & -5.3 & -3.8 & -8.6 & -0.5 & -1.4 \\ -0.5 & -0.5 & -4.1 & -1.0 & -5.1 & -7.7 \\ -1.9 & -1.9 & -9.7 & -3.9 & -11.7 & -19.5 \end{bmatrix} \qquad \Delta \boldsymbol{f} = \begin{bmatrix} -4.5 \\ -6.3 \\ -59.3 \\ -7.6 \\ -8.7 \\ -22.7 \end{bmatrix}$$

$$\Delta \boldsymbol{x} = \begin{bmatrix} -21.7 \\ -21.2 \\ -170.0 \\ -34.9 \\ -27.6 \\ -71.3 \end{bmatrix} \qquad \Delta \boldsymbol{v} = \begin{bmatrix} 0.0 \\ 0.0 \\ -109.1 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

The supply-driven method yields slightly lower adjustments of the EU compared to the demand-driven approach. Intermediate good exports from Russia to the EU decline relatively less, inducing a lower drop in European total output of about 0.70%.

5 Application

In a first exercise we try to model the energy supply reduction from Russia to the European Union in the aftermath of the Russian invasion in Ukraine. Therefore, we use the most recent global Input-Output Table from the OECD (ICIO).[10] The ICIO features 66 countries plus a Rest-of-the-World aggregate and 45 unique industries based on the latest UN industry classification.[11,12]

In order to model the energy supply reduction from Russia to the EU we employ the following scenarios:

1. Leontief Demand Driven
    - 50% reduction to final demand of Russian energy[13] in the 'big five'[14]
    - 20% reduction to final demand of Russian energy producing products[15] in the 'big five'
2. Ghosh Supply Driven
    - 50% reduction to value added of the Russian energy sector
    - 20% reduction to value added of the Russian energy producing products sector
3. Hypothetical Extraction
    - 50% reduction to output of the Russian energy sector

[9] All matrices are in the original order.

[10] OECD (2021), OECD Inter-Country Input-Output Database, http://oe.cd/icio

[11] International Standard Industrial Classification of All Economic Activities (ISIC) Revision 4, https://unstats.un.org/unsd/classifications/Family/Detail/27

[12] See Table 5 and 6 in the appendix for a full country and industry list.

[13] Industry code and name: D35, 'Electricity, gas, steam and air conditioning supply'

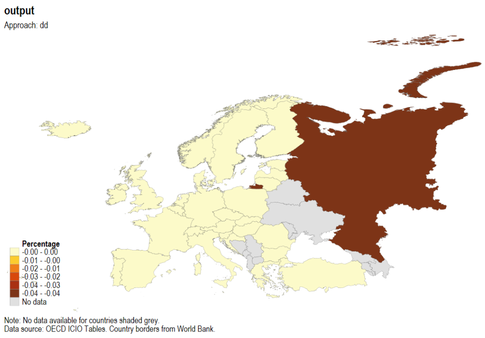
[14] Germany, France, Italy, Spain and the Netherlands

[15] Industry code and name: D05T06, 'Mining and quarrying, energy producing products'

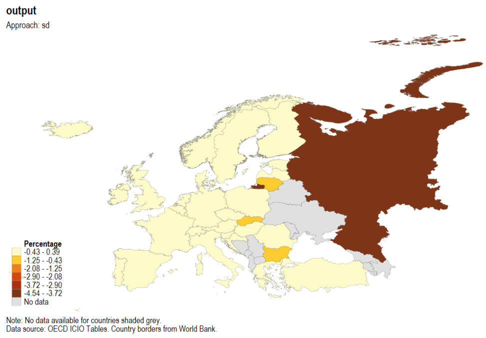
- 20% reduction to output of the Russian energy producing products sector

Figure 2 displays the percentage change in total output per country associated with the four methods. First, we notice Russia is by far more affected than EU countries across all methods.[16] Second, EU countries seem to be similarly affected. This is even the case for the Leontief Demand Driven model, where we apply a shock only to the 'big five'. This showcases the importance of production linkages across countries. Even though some EU countries are hit more than others in a first round, the tight EU production network transmits the shocks across the EU, such that in the end all EU countries are very similarly affected.

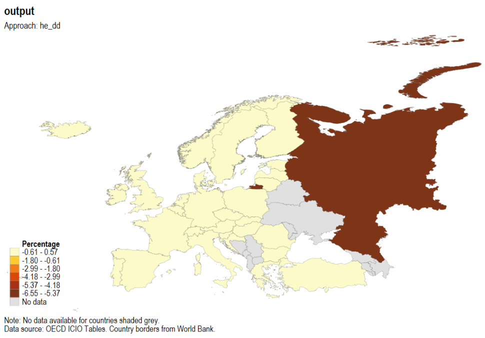
Figure 2: Country Output



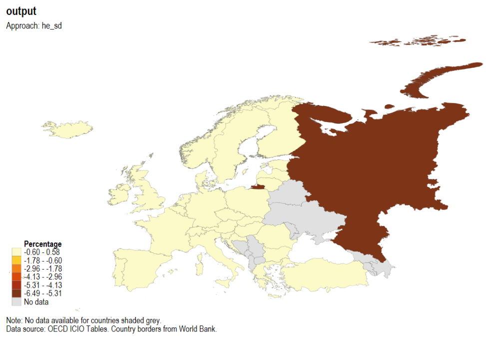
(a) Leontief Demand Driven



(b) Ghosh Supply Driven



(c) Hypothetical Extraction: Demand Driven



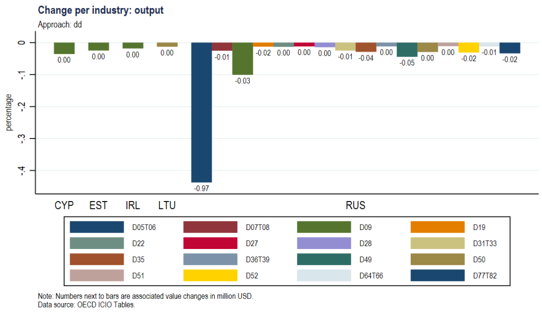
(d) Hypothetical Extraction: Supply Driven

Notes: The darker the shading the larger the magnitude of the percentage change.

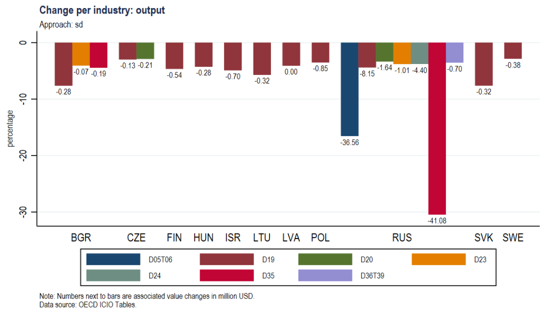
Each country-industry pair adjusts differently to shocks. Figure 3 displays the output change of the 20 most affected industries.

[16]See Figure 4 in the appendix for results without Russia to better highlight heterogeneous effects across the EU.

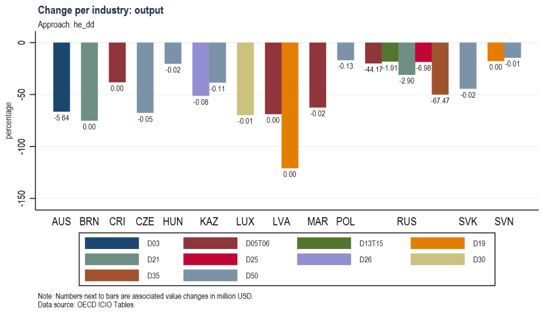
Figure 3: Industry Output (Top 20)



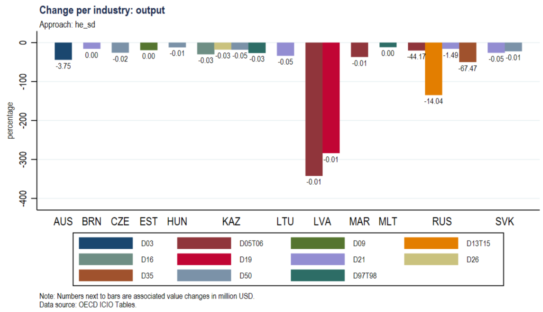
(a) Leontief Demand Driven



(b) Ghosh Supply Driven



(c) Hypothetical Extraction: Demand Driven



(d) Hypothetical Extraction: Supply Driven

Notes: Colors correspond to different industries. The x-axis shows the associated country. The y-axis shows percentage changes. The number next to the bar shows absolute changes in million dollars. Raking based on percentage change.

Similar figures can be readily produced for a vast number of variables, in particular all variables we have seen in previous sections, e.g. value added, exports and imports. Furthermore, based on our basic matrices we can also construct more elaborate variables, for example, domestic or foreign value added content of country-industry exports.

6 Conclusion

This paper discusses and illustrates the use of Input-Output Analysis to model economic shocks. The main advantage of Input-Output Analysis is that it incorporates itra- and international industry linkages and thereby, takes into account the network structure of international production and trade. In essence, these linkages translate to output multipliers which govern how a new equilibrium is attained. Table 4 collects the output multipliers associated with the four different methodologies.

Table 4: Input-Output Model Multipliers

	Leontief Demand-Driven (*backward linkages*)	Ghosh Supply-Driven (*forward linkages*)
Standard	$\boldsymbol{L} = (\boldsymbol{I} - \boldsymbol{A})^{-1}$	$\boldsymbol{G} = (\boldsymbol{I} - \boldsymbol{B})^{-1}$
Hypothetical Extraction	$(\boldsymbol{I} - \boldsymbol{A}_{ii})^{-1}\, \boldsymbol{A}_{ik}$	$\boldsymbol{B}_{ik}\, (\boldsymbol{I} - \boldsymbol{B}_{ii})^{-1}$

Each methodology has its advantages and disadvantages and is more or less suitable to capture different types of economic shocks. Therefore, the 'Simulation Tool' should be used to give a first estimate about the range of potential consequences for the economy.

Future work should be directed towards the following:

1. General equilibrium model with substitution (Caliendo and Parro, 2015)
    - allows for substitution of inputs across industries and trading partners for a more accurate upper and lower bound of estimates
    - allows for modelling of shocks via trade costs, output can then be used for the current methodology
    - allows to calculate welfare effects
2. Simultaneous demand and supply shocks (Pichler and Farmer, 2022)
3. Regional Input-Output Tables for more precise estimates for Flanders (see work by Prof. Magerman[17])
4. Environmental multipliers to better understand effects on CO2 emissions, energy and water usage
5. Employment multipliers to better understand effects on employment

[17] https://www.glennmagerman.com/research

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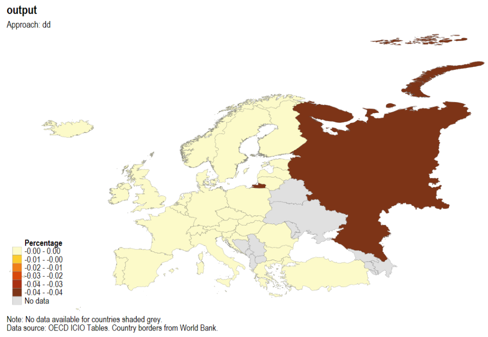
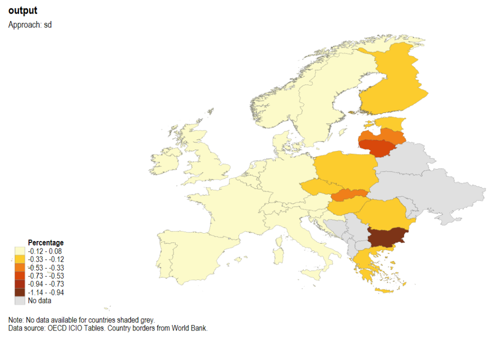
A Appendix

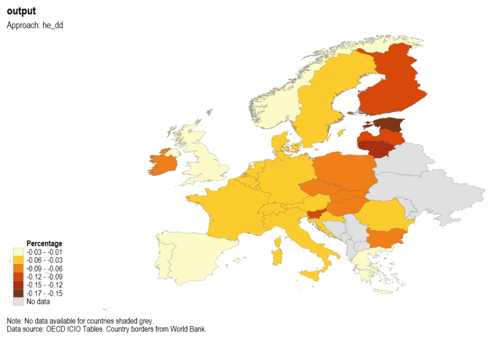
Figure 4: Country Output



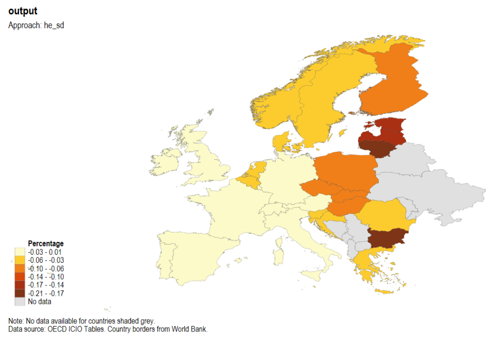
(a) Leontief Demand Driven



(b) Ghosh Supply Driven



(c) Hypothetical Extraction: Demand Driven



(d) Hypothetical Extraction: Supply Driven

Notes: The darker the shading the larger the magnitude of the percentage change. There is too little variation for the 'Leontief Demand Driven' model, therefore, we use the original figure.

Table 5: Country Information

ISO3 Code	English	Dutch	ISO3 Code	English	Dutch
	OECD Countries			**Non-OECD Countries**	
AUS	Australia	Australië	ARG	Argentina	Argentinië
AUT	Austria	Oostenrijk	BRA	Brazil	Brazilië
BEL	Belgium	België	BRN	Brunei	Brunei
CAN	Canada	Canada	BGR	Bulgaria	Bulgarije
CHL	Chile	Chili	KHM	Cambodia	Cambodja
COL	Colombia	Colombia	CHN	China	China
CRI	Costa Rica	Costa Rica	HRV	Croatia	Kroatië
CZE	Czech Republic	Tsjechië	CYP	Cyprus	Cyprus
DNK	Denmark	Denemarken	IND	India	Indië
EST	Estonia	Estland	IDN	Indonesia	Indonesië
FIN	Finland	Finland	HKG	Hong Kong	Hongkong
FRA	France	Frankrijk	KAZ	Kazakhstan	Kazachstan
DEU	Germany	Duitsland	LAO	Lao	Laos
GRC	Greece	Griekenland	MYS	Malaysia	Maleisië
HUN	Hungary	Hongarije	MLT	Malta	Malta
ISL	Iceland	IJsland	MAR	Morocco	Marokko
IRL	Ireland	Ierland	MMR	Myanmar	Myanmar
ISR	Israel	Israël	PER	Peru	Peru
ITA	Italy	Italië	PHL	Philippines	Filippijnen
JPN	Japan	Japan	ROU	Romania	Roemenië
KOR	Korea	Korea	RUS	Russian Federation	Rusland
LVA	Latvia	Letland	SAU	Saudi Arabia	Saudi-Arabië
LTU	Lithuania	Litouwen	SGP	Singapore	Singapore
LUX	Luxembourg	Luxemburg	ZAF	South Africa	Zuid-Afrika
MEX	Mexico	Mexico	TWN	Taiwan	Taiwan
NLD	Netherlands	Nederland	THA	Thailand	Thailand
NZL	New Zealand	Nieuw-Zeeland	TUN	Tunisia	Tunesië
NOR	Norway	Noorwegen	VNM	Viet Nam	Vietnam
POL	Poland	Polen	ROW	Rest of the World	Anderen
PRT	Portugal	Portugal			
SVK	Slovak Republic	Slovakije			
SVN	Slovenia	Slovenië			
ESP	Spain	Spanje			
SWE	Sweden	Zweden			
CHE	Switzerland	Zwitserland			
TUR	Turkey	Turkije			
GBR	United Kingdom	Verenigd Koninkrijk			
USA	United States	Verenigde Staten			

Table 6: Industry Codes and Labels

OECD Code	OECD Label	ISIC Code
D01T02	Agriculture, hunting, forestry	01, 02
D03	Fishing and aquaculture	03
D05T06	Mining and quarrying, energy producing products	05, 06
D07T08	Mining and quarrying, non-energy producing products	07, 08
D09	Mining support service activities	09
D10T12	Food products, beverages and tobacco	10, 11, 12
D13T15	Textiles, textile products, leather and footwear	13, 14, 15
D16	Wood and products of wood and cork	16
D17T18	Paper products and printing	17, 18
D19	Coke and refined petroleum products	19
D20	Chemical and chemical products	20
D21	Pharmaceuticals, medicinal chemical and botanical products	21
D22	Rubber and plastics products	22
D23	Other non-metallic mineral products	23
D24	Basic metals	24
D25	Fabricated metal products	25
D26	Computer, electronic and optical equipment	26
D27	Electrical equipment	27
D28	Machinery and equipment, nec	28
D29	Motor vehicles, trailers and semi-trailers	29
D30	Other transport equipment	30
D31T33	Manufacturing nec; repair and installation of machinery and equipment	31, 32, 33
D35	Electricity, gas, steam and air conditioning supply	35
D36T39	Water supply; sewerage, waste management and remediation activities	36, 37, 38, 39
D41T43	Construction	41, 42, 43
D45T47	Wholesale and retail trade; repair of motor vehicles	45, 46, 47
D49	Land transport and transport via pipelines	49
D50	Water transport	50
D51	Air transport	51
D52	Warehousing and support activities for transportation	52
D53	Postal and courier activities	53
D55T56	Accommodation and food service activities	55, 56
D58T60	Publishing, audiovisual and broadcasting activities	58, 59, 60
D61	Telecommunications	61
D62T63	IT and other information services	62, 63
D64T66	Financial and insurance activities	64, 65, 66
D68	Real estate activities	68
D69T75	Professional, scientific and technical activities	69 to 75
D77T82	Administrative and support services	77 to 82
D84	Public administration and defence; compulsory social security	84
D85	Education	85
D86T88	Human health and social work activities	86, 87, 88
D90T93	Arts, entertainment and recreation	90, 91, 92, 93
D94T96	Other service activities	94, 95, 96
D97T98	Activities of households as employers; activities of households for own use	97, 98